

## OPINION

# Should we teach general relativity in high school? Why and how?

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## Abstract

We discuss the proposal of teaching General Relativity and its spin-offs (black holes, gravitational waves, cosmology) as a part of the High School curriculum, a task already undertaken by a few countries. We point out the importance of this proposal for the students and some possible reasons for the resistance of educators and scientists. A suggestion about how to circumvent the well-known mathematical and conceptual difficulties associated with this formidable task is made. It is not enough to stay at a qualitative level of explanation for High School students, although it would be desirable and positive at younger ages.

**Keywords:** Teaching General Relativity; Mathematical content; Secondary School

## 1 General Relativity, "Modern Physics" and Teaching

A strong argument for the 21st century as "the Century of Gravitation" can be made based on the advances in theoretical/observational Cosmology (Daniel et al., 2010; Ishak, 2019), study and imaging of real black holes (Akiyama et al., 2019; Eckart et al., 2017) (including the Nobel Prize that was recently awarded to three scientists for their work on the supermassive black hole at the Milky Way center (Schirber, 2020), discovery of gravitational waves (Abbott et al., 2016) and other important developments (Joshi and Malafarina, 2011; Kochanek et al., 2001; McKee and Ostriker, 2007; Metcalf et al., 2019). These wonderful facts, with frequent presence in the media, are unified by their fundamental Relativistic character, that is, Newtonian gravitation is not enough to support and explain the new findings. Our main point in this short Opinion text is that while it may be enough to introduce some qualitative material related to gravitation before High School, it is imperative to bring some quantitative/formal approach for High School students for a number of reasons including the very nature of Sciences. In the long run, we believe that a qualitative, conceptual approach leads to the loss of a

fundamental connection with other branches of science. We elaborate on this point below.

It is no secret that under the time-dependent name of "Modern Physics", still widely used in schools and introductory university courses, groups a bunch of disciplines that are a century old or so. The lag between the educational content in hard sciences and the frontier knowledge shared by the scientists is enormous and has been put at the center of the debate many times (Goldader, 2002; Pasachoff, 2002). The rise of General Relativity (GR) as a concrete frontier topic adds a new twist to an old problem: *What should we teach? and how?*

Several colleagues would certainly agree with attempts to "close the gap". As a matter of fact, there are a handful of concrete initiatives to introduce these topics to catch up with the research news and give the students a broader perspective of science and the knowledge of the world. South Korea, Norway, Scotland (Kersting et al., 2018) are nationally engaged, and other countries like Australia feature Centres of Excellence and people dedicated to follow this path (Blair et al., 2016). We concretely target High School students, mature enough to be attained by quantitative and conceptual arguments, some of them possibly on their way to a career in science. In this sense, the material

developed for younger students is very valuable since each age group needs a different instructional approach, and a qualitative one would be excellent for them.

Closing this gap is definitely important, but not only for "catching up" purposes. For a High School student to have a real glimpse of some GR concepts, as well as their applicability, could be of extraordinary and unparalleled value for his/her genuine interest in science, even for those that will be heading to different activities.

Here, we are not talking about a highly non-trivial spacetime metric and, say, "non-ideal" energy-momentum tensor, to compute the Einstein tensor components, derive the Einstein field equations and even solve them for all variables involved. We are, for example, encouraging the teaching of the metric (tensor) first notions, perhaps by quoting that the metric tensor is what rules the measuring of distances in curved spaces. The introduction of Minkowski metric is very welcome as an elegant shortcut to Euclidian space.

We also believe a deep approach of Einstein's equivalence principle is wholesome for High School students, bringing to students the possibility of wondering about distinguishing between "purely acceleration" and gravitation effects is certainly of great value. This could also prompt them to search for a profound understanding of coordinate systems and coordinate transformations. Finally, the tidal forces calculation should inform the learner if they are dealing with a false (due simply to coordinate transformations) or genuine gravitational field.

A closer examination to the material and topics presented to the students about GR gravitation reveals that the vast majority is of qualitative character, and also that new approaches to the education (like the use of cartoons, videos, etc.) are widely used for this task. It seems that there is a consensus on the difficulties of concepts of GR topics, and an implicit recognition that a formal approach is out of question. In other words, that "the mathematics is not there till we put it there" (A.S. Eddington) and that concepts precede the mathematical description. We face here a first problem: many "hard" scientists would feel that the essential part of gravitation is lost without any mathematical formulation. In fact, this can be traced back to an old resonance among scientists of Galileo's words:

*"The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word." (G. Galilei, Il Saggiatore, p.175)*

and the very evolution of hard sciences as well, which can be qualified as "hermetical", that is, need preparation on specific tools and ideas to advance.

In fact, the relation of mathematics and the rest of "hard sciences" seems to be based to a good extent on a kind of symbiosis, in which the formal mathematical formulation is not optional, but rather is a part of the *logos* (McDonnell, 2017). These kinds of beliefs are in fact the basis of modern science curricula for hard sciences. While this has never been a problem, for example, for biological sciences, the "softening" of physics and astronomy, even with the purpose of making accessible difficult subjects, would prompt some degree of rejection among the practitioners. As an example, it is clear that the survival of basic topics in High School curricula is based on the conviction that they are important, a doorway to more complicated things, but also on the fact that they are "doable" mathematically, even if restricted to simple exercises and problems, for example, kinematics, thermal physics and other well-known topics.

We believe that if teaching GR implies giving up mathematics as part of the *logos*, this would prompt a long-term change in the very concept of hard sciences, something seen as undesirable

to say the least. It is not too early to pretend an exposure of High School students to a set of carefully designed quantitative topics related to GR.

## 2 Which is the path forward?

Having gauged the situation of gravitational physics, we must address the question of *what to do with education?*. There are a number of resources not yet explored for the construction of a viable approach to GR and other difficult subjects, in which not only the concepts but also the formal formulations are difficult, such as quantum mechanics. Recently, the application of Bruner's ideas (Bruner, 2003) to a narrative contribution to GR has been suggested (Cardoso and Gurgel, 2013), as identified in Einstein's methodology. *Gedankenexperiments* are quite related to this task. In addition, the exploration of a less formal approach to the mathematical content present, for example in the famous work by Misner, Thorne and Wheeler (2017)2017. We believe this latter approach, when properly adapted to the High School level, could be a bridge between purely conceptual teaching and a kind of "math without math", pointed to recover the very essence of the symbiosis between physics and mathematics.

The whole idea is a kind of "verbal mathematics" first, aimed to illuminate the deep meaning of relations between physical quantities. An example is Newton's Law written as:

$$\text{variation of velocity} \propto \text{force} \quad (1)$$

followed by a proper discussion and formalization leading to  $\vec{F} = m \times \vec{a}$  that is, giving time and tools to grasp a more abstract form of physical reasoning through mathematics. Such a path can be also followed towards GR expressions, such as the renowned field equations written as:

$$\text{curvature of spacetime} \propto \text{distribution of matter/energy} \quad (2)$$

at least those involving scalar and vector quantities. Of course, in this case it is mandatory for the students to previously perceive the space-time concept and its non-absolute character, possibly through the well-known rubber sheet experiment (one can also get in touch with Reference (Stachel, 2005)), stressing to the students the difficulty in curving space with a simple numerical estimate. Overall, a kind of "computational thinking" would be developed by students and the main ideas will be encoded by some symbols, ultimately, this is what mathematics is all about.

The connection with "Newtonian analogues" could also be explored. In fact, a "quasi-Newtonian" approach would be an important step forward for a variety of important topics, such as black holes and gravitational waves. For example, Friedmann equations were written in a Newtonian mood (Misner et al., 2017, p. 708). The quasi-Newtonian derivation of the Schwarzschild radius  $R = \frac{2GM}{c^2}$  (Visser, 2005) is another example. Finally, we note that well-suited material, involving pre-calculus and geometry targeted to these tasks exists (Kraus and Zahn, 2018; Schutz, 2003; Zahn and Kraus, 2014, 2018) and would be an excellent starting point to conduct large-scale educational research aimed to test how it leads to significant learning (Ausubel, 2000) and what can be done to improve its efficiency in the classroom.

In short, it is clear that a long-term investigation of this possibility and its testing with professors and students is ahead and, in many cases, already available, a feasible yet involved pedagogical strategic project with ample applicability. The important point to be stressed again is not to give up a minimal degree of formalization without trying.

As a final statement, we strongly agree with the introduction of General Relativity (and other topics) in High School, but believe that a qualitative, conceptual approach only causes a loss of a fundamental connection with other branches of science. It is sometimes observed that a procedure like this is enough to "separate", for example, astronomy from physics, as if fundamentally different things. Is it enough to teach astronomy and physics qualitatively than nothing at all? Yes, at least in elementary school, but we should also attempt to develop a viable approach suited to this new challenge for the High School level. This is possible and would be important to retain the hard science character of these disciplines, while also reinforcing the unity of sciences in a concrete fashion. A hard science taught "without mathematics" is not an achievement we should be proud of.

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